order of magnitude, Continuity

orders of magnitude: which grows faster?

$\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$ $0<L<\infty$, f & g grow at the same rate

$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$, f grows at a slower rate than g

$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$, f grows faster than g

Mar 10-1:06 PM

$f(x)=x^3$ $g(x)=\sqrt{x^6+x^2}$ - compare with $\sqrt{x^6}$

x^3

$\lim_{x\to\infty} \frac{x^3}{\sqrt{x^6+x^2}} = 1$

like x^3

f & g same

Mar 10-1:11 PM

Continuity

informal def. draw graph without lifting pen
no holes, asymptotes, gaps

continuity at a point

f(x) is continuous at $x=c$

if.
1. $\lim_{x \to c} f(x)$ exists rhl = lhl
2. $f(c)$ exists
3. #1 = #2

continuity on an interval

f(x) is continuous on [a,b] if
it is continuous at every point in [a,b]

Mar 10-1:13 PM

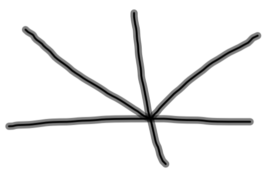
is f(x) continuous ? implied interval is the domain

if f(x) is differentiable then its continuous

converse may not be true

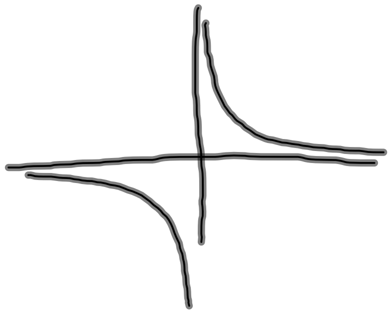
if f(x) is continuous ... it may or may not be differentiable

$y=|x|$



Mar 10-1:19 PM

Is $y = \frac{1}{x}$ continuous? Yes



not continuous at $x=0$
but 0 not in domain

Is $y = \frac{\sin x}{x}$ continuous on $(-\infty, \infty)$? No

hole at $x=0$ $y=1$

Mar 10-1:24 PM

continuous extension of $y = \frac{\sin x}{x}$

$$y = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1 & x = 0 \end{cases}$$

Mar 10-1:31 PM

Let $f(x) = \begin{cases} 2x+1, & x \le 2 \\ \frac{1}{2}x^2 + k, & x > 2 \end{cases}$

What value of k will make $f(x)$ continuous at $x=2$? Justify your answer

$k = 3$

1. $\lim_{x \to 2} f(x)$ exists

left / right

$\lim_{x \to 2^-} 2x+1 = 5$ $\lim_{x \to 2^+} \frac{1}{2}x^2 + k = 2+k$

$2 + k = 5$ $k = 3$

2. $f(2)$ exists: $f(2) = 5$

3. #1 = #2

Mar 10-1:33 PM